

It seems best therefore in this, as well as other cases, to omit the terms involving cosines of odd multiples of θ .

3. As regards the observations with the artificial surface in water, the coefficients of the cosines in the expression for the azimuths and of the sines in the expression for the polarising angles are insensibly small, indicating no introduction of asymmetry with respect to the principal plane arising from the process of polishing. The coefficients of the cosines of odd multiples of θ in the second expression are also insensible. The constant term in the first expression, representing (on the assumption of symmetry with respect to the principal plane) the index error of the circle carrying the Nicol, agrees almost exactly with those obtained for the cleavage surfaces in air and water.

It would seem best then to omit those terms which we have reason to think are really *nil*, and which the observations show to be at any rate extremely small, and to exhibit the final result accordingly.

February 11, 1886.

Professor STOKES, D.C.L., President, in the Chair.

The Presents received were laid on the table, and thanks ordered for them.

The following Papers were read:—

- I. “On the Theory of Lubrication and its Application to Mr. Beauchamp Tower’s Experiments, including an Experimental Determination of the Viscosity of Olive Oil.” By Professor OSBORNE REYNOLDS, LL.D., F.R.S. Received December 29, 1885.

(Abstract.)

Lubrication, or the action of oils and other viscous fluids to diminish friction and wear between solid surfaces, does not appear to have hitherto formed a subject for theoretical treatment. Such treatment may have been prevented by the obscurity of the physical actions involved, which belong to a class as yet but little known, namely, the boundary or surface actions of fluids; but the absence of such treatment has also been owing to the want of any general laws revealed by experiment.

The subject is of such fundamental importance in practical mechanics, and the opportunities of observation so frequent, that it

may well be a matter of surprise that any general laws should have for so long escaped detection.

Besides the general experience obtained, the friction of lubricated surfaces has been the subject of much experimental investigation by able and careful experimenters; but although in many cases empirical laws have been propounded, these fail for the most part to agree with each other and with the more general experience.

The most recent investigation is that of Mr. Tower, undertaken at the instance of the Institute of Mechanical Engineers. Mr. Tower's first report was published in November, 1883, and his second in 1884 ("Proc. Inst. Mechanical Engineers").

In these reports Mr. Tower, making no attempt to formulate, states the results of experiments apparently conducted with extreme care, and under very various and well-chosen circumstances. Those of the results which were obtained under the ordinary conditions of lubrication so far agree with the results of previous investigators as to show a want of any regularity. But one of the causes of this want of regularity, viz., irregularity in the supply of lubricant, appears to have occurred to Mr. Tower early in his investigation, and led him to include amongst his experiments the unusual circumstance of surfaces completely immersed in an oil-bath. This was very fortunate, for not only do the results so obtained show a great degree of regularity, but while making these experiments he was accidentally led to observe a phenomenon which, taken with the results of his experiments, amounts to a crucial proof that in these experiments with the oil-bath the surfaces were completely and continuously separated by a film of oil; this film being maintained by the motion of the journal, although the pressure of the oil at the crown of the bearing was shown by actual measurement to be as much as 625 lbs. per square inch above the pressure in the oil-bath.

These results with the oil-bath are very important, notwithstanding that the condition is not common in practice. They show that with perfect lubrication a definite law of the variation of the friction with the pressure and the velocity holds for a particular journal and brass. This strongly implies that the irregularity previously found was due to imperfect lubrication. Mr. Tower has brought this out. Substituting for the bath an oily pad of tow pressed against the free part of the journal, and making it so slightly greasy that it was barely perceptible to the touch, he again found considerable regularity in the results. These were, however, very different from those with the bath. Then with intermediate lubrication he obtained intermediate results, of which he says: "Indeed, the results, generally speaking, were so uncertain and irregular that they may be summed up in a few words. The friction depends on the quantity and uniform distribution of the oil, and may be anything between the oil-bath results

and seizing, according to the perfection or imperfection of the lubrication."

On reading Mr. Tower's first report, it occurred to the author that in the case of the oil-bath the film of oil might be sufficiently thick for the unknown boundary actions to disappear, in which case the results would be deducible from the equations of hydrodynamics.

Mr. Tower appears to have considered this, for he remarks that, according to the theory of fluid friction the resistance would be as the square of the velocity, whereas in his results it does not increase according to this law.

Considering how very general the law of resistance as the square of the speed is with fluids, there is nothing remarkable in it being assumed to hold in such a case. But the study of the behaviour of fluid in very narrow channels, and particularly the recent determination by the author of the critical velocity at which the law changes from that of the square of the velocity to that of the simple ratio, shows that with such highly viscous fluids as oils, such small spaces as those existing between the journal and its bearing, and such limited velocities as that of the surface of the journal, the resistance would vary, *cæteris paribus*, as the velocity. And further, the thickness of the oil film would not be uniform, and might be affected by the velocity, and as the resistance would vary, *cæteris paribus*, inversely as the thickness of the film, the velocity might exert in this way a secondary influence on the resistance; and further still, the resistance would depend on the viscosity (commonly called the body) of the oil, and this depends on the temperature. But as Mr. Tower had been careful to make all his experiments in the same series with something at a temperature of 90° F. (he does not state precisely what), it did not at first appear that there could be any considerable temperature effect in his results.

The application of hydrodynamical equations for viscous fluids to circumstances similar to those of a journal and a brass in an oil-bath, in so far as they are known, at once led to an equation* between the variation of pressure over the surface and the velocity, which appeared to explain the existence of the film of oil at high pressure.

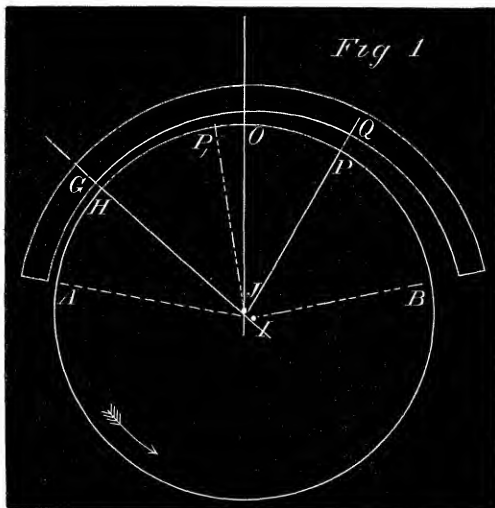
This equation was mentioned in a paper read before Section A at the British Association, at Montreal, 1884. It also appears from a paragraph in the Presidential Address (p. 14, Brit. Assoc. Rep., 1884)

$$* \frac{dp}{dx} = \frac{6\mu U(h-h_1)}{h^3} \quad \text{No. of equation in the paper.} \quad (31)$$

in which p is the intensity of pressure, μ coefficient of viscosity, x the direction of motion, h the interval between the journal and the brass, h_1 being the value of h for which the pressure is a maximum, U the surface velocity in the direction of x .

that Prof. Stokes and Lord Rayleigh had simultaneously arrived at a similar result. At that time the author had no idea of attempting the integration of this equation. On subsequent consideration, however, it appeared that the equation might be so transformed* as

FIG. 1.



* If the journal and brass are both of circular section, as in fig. 1, and R is the radius of the journal, $R+a$ radius of brass, J the centre of the journal, I the centre of the brass, $JI=ca$, HG the shortest distance across the film, IO the line of loads through the middle of the brass, A the extremity of the brass on the off side, B on the on side, P_1 the point of greatest pressure,

Putting

$$OIH = \phi_0 - \frac{\pi}{2}$$

$$OIP_1 = \phi_1$$

$$OIP = \theta$$

$$h = a \{ 1 + c \sin (\theta - \phi_0) \}$$

$$h_1 = a \{ 1 + c \sin (\phi_1 - \phi_0) \}$$

the equation (31) becomes

$$\frac{dp}{d\theta} = \frac{6R\mu c \{ \sin (\theta - \phi_0) - \sin (\phi_1 - \phi_0) \}}{a^2 \{ 1 + c \sin (\theta - \phi_0) \}^3} \quad \dots \dots (48)$$

If $\frac{a}{R}$ is small. This equation, which is at once integrable when c is small, has been integrated by approximation when c is as large as 0.5.

The friction is given by an equation

$$f = -\frac{1}{2}R \frac{dp}{d\theta} a \{ c \sin (\theta - \phi_0) \} - \mu \frac{U_1 - U_0}{a \{ 1 + c \sin \theta - \phi_0 \}} \quad \dots \dots (49)$$

This is also approximately integrated up to $c=0.5$.

to be approximately integrated by considering certain quantities small, and the theoretical results thus definitely compared with the experimental.

The result of this comparison was to show that with a particular journal and brass the mean thickness of the film would be sensibly constant for all but extreme values of load divided by the viscosity, and hence if the coefficient of viscosity were constant the resistance would increase approximately as the speed.

As this was not in accordance with Mr. Tower's experiments, in which the resistance increased at a much slower rate, it appeared that either the boundary actions became sensible, or that there must be a rise in the temperature of the oil which had escaped the thermometer used to measure the temperature of the journal.

That there would be some excess of temperature in the oil film on which all the work of overcoming friction is spent is certain, and after carefully considering the means of escape of this heat, it appeared probable that there would be a difference of several degrees between the oil-bath and the film of oil.

This increase of temperature would be attended with a diminution of viscosity, so that as the resistance and temperature increased with the velocity there would be a diminution of viscosity, which would cause the increase of the resistance with the velocity to be less than the simple ratio.

In order to obtain a quantitative estimate of these secondary effects, it was necessary to know the exact relation between the viscosity of the oil and the temperature. For this purpose an experimental determination was made of the viscosity of olive oil at different temperatures as compared with the known viscosity of water. From the result of these experiments an empirical formula has been deduced

ϕ_0 and ϕ_1 and c have to be determined from the conditions of equilibrium, which are

$$\int_{-\theta_1}^{\theta_1} \{p \sin \theta - f \cos \theta\} d\theta = 0 \quad (44)$$

$$\int_{-\theta_1}^{\theta_1} \{p \cos \theta + f \sin \theta\} d\theta = \frac{L}{R} \quad (45)$$

$$\int_{-\theta}^{\theta} f d\theta = \frac{M}{R^2} \quad (46)$$

where $2\theta_1$ is the angle subtended by the brass, L the load, and M the moment of friction.

The solution of these equations may be accomplished when c is small and has been approximately accomplished for particular values of c up to 0.5, the boundary conditions as regards p being

$$\theta = \pm \theta_1 \quad p = p_0,$$

whence substituting the values of ϕ_1 , ϕ_0 , c in (48) and (49), and integrating, the values of the friction and values of the pressure are obtained.

used to keep the brass in its place, or by any irregularity in the longitudinal section of the journal or brass. Although no direct reference is made to such flanges in Mr. Tower's reports, it is such a common custom to neck the shaft to form the journal that there is great probability of the flanges being used. A coefficient n has therefore been introduced into the theory, which includes both the effect of necking and of irregularity in longitudinal section. Giving n the value 1.25, the calculated results came into accordance with all Mr. Tower's results for olive oil, the difference being such as might well be attributed to experimental inaccuracy, and this both as regards the frictions measured with one brass, No. 1, and the distribution of the pressure round the journal with another, No. 2.

Not only does the theory thus afford an explanation of the very novel phenomena of the pressure in the oil film, but it also shows, what does not appear in the experiments, how the various circumstances under which the experiments have been made affect the results.

Two circumstances in particular which are brought out as principal circumstances by the theory seem to have hitherto entirely escaped notice, even that of Mr. Tower.

One of these is a , the difference in the radii of the journal and of the brass or bearing. It is well known that the fitting between the journal and its bearing produces a great effect on the carrying power of the journal, but this fitting is supposed to be rather a matter of smoothness of surface than a degree of difference in radii. The radius of the bearing must always be as much larger than that of the journal as is necessary to secure an easy fit, but more than this does not seem to have been suggested.

It now appears from this theory that if viscosity were constant the friction would be inversely proportional to the difference in the radii of the bearing and journal, and this although the arc of contact is less than the semi-circumference; and taking temperature into account it appears from the comparison of the theoretical frictions with the experiment on brass No. 1, that the difference in the radii at 70° F. was

$$a=0.00077 \text{ (inch),}$$

and comparing the theoretical pressures with those measured with brass No. 2,

$$a=0.00084 \text{ (inch),}$$

or the difference was 9 per cent. greater in the case of brass No. 2.

These two brasses were probably both bedded to the journal in the same way, and had neither been subjected to any great amount of wear, so that there is nothing surprising in their being so nearly the

same fit. It would be extremely interesting to find how far under continuous running prolonged wear tends to preserve this fit. Mr. Tower's experiments afford only slight indication of this. It does appear, however, that the brass expanded with an increase of temperature, and that its radius increases as the load increases in a very definite manner.

Another circumstance brought out by this theory, and remarked on both by Lord Rayleigh and the author at Montreal, but not before suspected is, that the point of nearest approach of the journal to the brass is not by any means in the line of load, and what is still more contrary to common supposition, it is on the *off** side of this line.

This point H moves as the ratio of load to velocity increases; when this ratio is zero, the point H coincides with O, then as the load increases it moves away to the left, till it reaches a maximum distance $\frac{1}{2}\pi - \phi_0$, being nearly $-\frac{\pi}{2}$. The load is still small, smaller than anything in Mr. Tower's experiments, even with the highest velocities. For further increase of load, H returns towards O, or $\frac{1}{2}\pi - \phi_0$ increases with the largest loads and smallest velocities to which the theory has been applied; this angle is about 40° . With a fairly loaded journal well lubricated it would thus seem that the point of nearest approach of brass to journal, *i.e.*, the centre of wear, would be about the middle of the *off* side of the brass.

This circumstance, the reason of which is rendered perfectly clear by the conditions of equilibrium, at once explains a singular phenomenon, incidentally pointed out by Mr. Tower, *viz.*, that the journal having been run in one direction for some time, and carrying its load without heating, on being reversed began to heat again, and this after many repetitions always heating on reversal, although eventually this tendency nearly disappeared. Mr. Tower's suggested explanation appears to the author as too hypothetical to be satisfactory, even in default of any other; and particularly as this is an effect which would necessarily follow in accordance with the theory, so long as there is wear. For the centre of wear, being on the *off* side of the line of loads, this wear will tend to preserve or diminish the radius of the brass on the *off* side, and enlarge it on the *on* side, a change which will, if anything, improve the condition for producing oil pressure while running in this direction, but which will damage the condition on which the production of pressure in the film depends when the journal is reversed and the late *off* side becomes the new *on* side. That with a well-worn surface there should be sufficient wear to produce this

* *On* and *off* sides are used by Mr. Tower to express respectively the sides of approach and recession, as B and A, fig. 1, p. 194, the arrow indicating the direction of motion.

result, with such slight amounts of using as those in Mr. Tower's experiments before reversal, seems doubtful, but supposing the brass new, and the surface more or less unequal, the wear for some time would be considerable, even after the initial tendency to heat had disappeared. Hence it is not surprising that the effect should have eventually seemed to disappear.

The circumstances which determine the greatest load which a bearing will carry with complete lubrication, *i.e.*, with the oil film continuous between brass and journal throughout the entire arc, are definitely shown in the theory, so long as the brass has a circular section.

As the ratio of the load to velocity increases HI or c increases, and the point H approaches O , when c reaches the value 0.5 , which makes $GH = a(1-c) = 0.5a$, the pressure of the oil in the film is everywhere greater than at A and B , the pressure in the bath, but for a further increase in the load the pressure falls near A on the *off* side, the fall will cause the pressure to become less than that of the atmosphere, or if sufficient to become absolutely negative, until discontinuity or rupture of the oil film occurs. The film will then only extend between brass and journal over a portion of the whole arc, and a smaller portion as the load increases or velocity diminishes, *i.e.*, as c increases. Thus since the amount of negative pressure which the oil will bear depends on circumstances which are uncertain, the limit of the safe load for complete lubrication is that which causes the least separating distance to be half the difference in radii of the brass and journal.

The rupture of the oil does not take place at the point of nearest approach, but on the *off* side of this, and will only extend up to a point P_2 definitely shown in this theory, which is at the same distance on the *off* side of H as P_1 is on the *on* side. Hence after this rupture the brass may still be in equilibrium, entirely separated from the journal, and the question as to whether it will carry a greater load without descending on to the journal will depend on the relative values of $\frac{a}{R}$ and on the smallness of the velocity. The condition then becomes the same as that for imperfect lubrication, and except in the case of a being very small, theory shows that the ultimate limit to the load will be the same with the oil-bath and with partial lubrication as Mr. Tower found it to be.

This much may be inferred without effecting the integrations for imperfect lubrications; could these be effected, the theory would be as applicable to partial lubrication as it has been to complete lubrication, *i.e.*, a sufficient supply of oil. And as it is, sufficient may be seen to show that with any supply of oil, however insufficient for complete lubrication, the brass will still be completely separated from the journal, although the supporting film of oil will not touch the brass

except over a limited area, and it is shown by general reasoning that in the one extreme, when the oil is very limited, the friction increases directly as the load, and is independent of the velocity, while in the other, where the oil is abundant, the circumstances are those of the oil-bath.

The effect of the limited length of bearings, and the escape of the oil at the ends, is also apparent in the equations.

Although in the main the present investigation has been directed to the circumstances of Mr. Tower's experiments, namely, a cylindrical journal revolving in a cylindrical brass, the main object has been to establish a general and complete theory based on the hydrodynamical equations for viscous fluids. Hence it has been thought necessary to proceed from the general equations, and to deduce the equations of lubrication in a general form, from which the particular form for application has been obtained. It has been found necessary also to consider somewhat generally the characters of fluid friction and viscosity.

The property of viscosity has been discussed in Section II of the paper, which section also contains the account of the experimental investigation as to the viscosity of olive oil. The general theory deduced from the hydrodynamical equations for viscous fluids with methods of application, including two cases besides the cylindrical journal in which the equations become completely integrable, viz., two plane surfaces of elliptical shape approaching, and one plane sliding over another not quite parallel plane surface, is given in Sections IV, V, VI, and VII.

The physical considerations of the effect of the heat generated are discussed in Section VIII.

As there are some circumstances which cannot be taken into account in the definite reasoning, particularly as regards incomplete lubrication, besides which, as the definite reasoning tends to obscure the more immediate purpose of the investigation, a preliminary discussion of the problem presented by lubrication, illustrated by aid of graphic methods, has been introduced as Section III.

Finally, the definite application to Mr. Tower's experiments is given in Section IX, which concludes as follows:—

The experiments to which the theory has been definitely applied may be taken to include all Mr. Tower's experiments with the 4-inch journal and oil-bath, in which the number of revolutions per minute was between 100 and 450, and the nominal loads in pounds per square inch between 100 and 415. The other experiments with the oil-bath were with loads from 415 till the journal seized at 520, 573, or 625, and a set of experiments with brass No. 2 at 20 revolutions per minute. All these experiments were under extreme conditions, for which by the theory c was so great as to render lubrication incom-

plete, and preclude the application of the theory without further integrations.

The theory has, therefore, been tested by experiments throughout the extreme range of circumstances to which the particular integrations undertaken are applicable, and the results, which in many cases check one another, are consistent throughout.

The agreement of the experimental results with the particular equations obtained on the assumption that the brass as well as the journal are truly circular, must be attributed to the same causes as the great regularity presented by the experimental results themselves.

Fundamental amongst these causes is, as Mr. Tower has pointed out, the perfect supply of lubricant obtained with the oil-bath. But nearly as important must have been the truth with which the brasses were first fitted to the journal, the smallness of the subsequent wear and the variety of the conditions as to magnitude of load, speed, and direction of motion.

That a brass in continuous use should preserve a circular section with a constant radius requires either that there should be no wear at all, or that the wear at any point P should be proportional to $\sin(90^\circ - \text{POH})$.

Experience shows that there is wear in ordinary practice, and even in Mr. Tower's experiments, there seems to have been some wear. In these experiments, however, there is every reason to suppose that the wear would have been approximately proportional to $c \sin(\phi_0 - \theta) = c \sin(90^\circ - \text{POH})$, because this represents the approach of the brass to the journal within the mean distance a , for all points except those at which it is negative, at these there would be either no wear at all or a slight positive wear. So long, then, as the journal ran in one direction only, the wear would tend to preserve the radius and true circular form of that portion of the arc from A to F (fig. 1, note *), altering the radius at F, and enlarging it from F to B. On reversal, however, A and F change sides, and hence alternate motion in both directions would preserve the radius constant all over the brass.

The experience emphasised by Mr. Tower, that the journal, after running for some time in one direction, would not run at first in the other, strongly bears out this conclusion. Hence it follows that had the journal been continuously run in one direction, the condition of lubrication, as shown by the distribution of oil pressure round the journal, would have been modified, the pressure falling between O and B on the *on* side of the journal, a conclusion which is borne out by the fact that in the experiments with brass No. 2, which was run for some time continuously in one direction, the pressure measured on the *on* side is somewhat below that calculated on the assumption of circular form, although the agreement is close for the other four points.

When the surfaces are completely separated by oil it is difficult to see what can cause wear. But there is generally metallic contact at starting, and hence abrasions, which will introduce metallic particles into the oil (blacken it), these particles will be more or less carried round and round with the journal, causing wear and increasing the number of metallic particles and the viscosity of the oil. Thus the rate of wear would depend on the metallic particles in the oil, the values of c , $\frac{1}{a}$, and the velocity of the journal, and hence would render the greatest velocity of the journal at which the maximum load with a large value of c could be carried, small; a conclusion which seems to be confirmed by Mr. Tower's experiments with brass No. 2 at twenty revolutions a minute.

In cases such as engine bearings the wear causes the radius of curvature of the brass continually to increase, and hence a and c must continually increase with wear. But, in order to apply the theory to such cases, the change in the direction of the load (or the velocity of approach of the surfaces) would have to be taken into account.

That the circumstances of Mr. Tower's experiments are not those of ordinary practice, and hence that the particular equations deduced in order to apply the theory definitely to these experiments do not apply to ordinary cases, does not show that the general theory as given in the general equation could not be applied to ordinary cases were the conditions sufficiently known.

These experiments of Mr. Tower have afforded the means of verifying the theory for a particular case, and hence have so far established its truth as applicable to all cases for which the integrations can be effected.

The circumstances expressed by—

$$\mu \frac{L}{U R} c_1 \phi_0 \phi_1 n_1 m c' A E B,$$

which are shown by the theory to be, together with the supply of lubricant, the principal circumstances on which lubrication depends, although not the same in other cases, will still be the principal circumstances, and indicate the conditions to be fulfilled in order to secure good lubrication.

The verification of the equations for viscous fluids under such extreme circumstances affords a severe test of the truth and completeness of the assumptions on which these equations are founded; and the result of the whole research is to point to a conclusion (important to natural philosophy) that not only in cases of intentional lubrication, but wherever hard surfaces under pressure slide over each other without abrasion, they are separated by a film of some foreign matter, whether perceptible or not; and that the question as to

whether the action can be continuous or not turns on whether the motion tends to preserve the foreign matter between the surfaces at the points of pressure, as in the almost if not quite unique case of the revolving journal, or tends to remove it, and sweep it on one side, as is the action of all backward and forward rubbing with continuous pressure.

The fact that a little grease will enable any surfaces to slide for a time has tended doubtless to obscure the action of the revolving journal to maintain the oil between the surfaces at the point of pressure, and yet, although only now understood, it is this action that has alone rendered machinery or even carriages possible. The only other self-acting system of lubrication is that of reciprocating motion with intermittent pressure and separation of the surfaces to draw the oil back or to draw a fresh supply. This is important in certain machinery, as in the steam-engine, and is as fundamental to animal mechanism as is the continuous lubricating action of the journal to mechanical contrivances.

- II. "The Electrical Phenomena accompanying the Process of Secretion in the Salivary Glands of the Dog and Cat." By W. MADDOCK BAYLISS, B.Sc., and J. ROSE BRADFORD, B.Sc., Senior Demonstrator of Anatomy in University College, London (from the Physiological Laboratory of University College). Communicated by E. A. SCHÄFER, F.R.S. Received February 4, 1886.

(Abstract.)

The glands examined were the submaxillary and parotid of the dog and cat, and in all of these we have determined that the process of secretion is accompanied by definite electrical changes; as, however, the submaxillary gland both in the dog and cat has been more thoroughly examined than the parotid, the present communication is confined almost entirely to the former.

The chorda tympani and sympathetic nerves were exposed in the usual manner, divided, and the peripheral ends arranged for stimulation, a canula being placed in Wharton's duct. The submaxillary gland having been exposed was led off in the following manner. One non-polarisable electrode was placed on the superficial or cutaneous aspect of the gland, and the second electrode so arranged in the wound as to touch the deep surface of the gland as close to the hilus as possible without pressing on the duct.

A Thomson galvanometer of high resistance was used.

Fig. 1.

